SOLUTION BY A. H. WILSON, Haverford College.

The discussion is for the center of the hexagon, an obvious position of equilibrium. Let this point be taken as origin, the hexagon placed with two vertices on the x-axis. Let r_0 be the side of the hexagon.

Form the potential function V. For one attracting center this is of the form $\int_{r_0}^{r} r^{-h}dV = r^{1-h}/(1-h) - r_0^{1-h}/(1-h)$. (The value $h = 1$ must be excepted.) For simplicity the origin is taken as a standard position.

$$V = \Sigma r^{1-h}/(1-h) - V_0.$$

Each r is of the form $[(x-s)^2 + (y-t)^2]^{1/2}$. Expand V in the neighborhood of the origin in powers of x and y. For r^{1-h} this expansion is

$$r^{1-h} = r_0^{1-h} + s(1-h)r_0^{-h-1}x + t(1-h)r_0^{-h-1}y + \tfrac{1}{2}[(1-h)r_0^{-h-1} - (1-h^2)s^2r_0^{-h-3}]x^2$$
$$- (1-h^2)r_0^{-h-3}st\cdot xy + \tfrac{1}{2}[(1-h)r_0^{-h-1} - (1-h^2)r_0^{-h-3}t^2]y^2 + \cdots.$$

For (s, t) substitute successively the coördinates of the hexagon points and sum:

$$V = V_0 - \tfrac{1}{2}(2+3h)r_0^{-h-1}(x^2+y^2) + \cdots.$$

In the neighborhood of the origin then the equations of motion are

$$m\frac{d^2x}{dt^2} = -\frac{\partial V}{\partial x} = (2+3h)r_0^{-h-1}x,$$

$$m\frac{d^2y}{dt^2} = -\frac{\partial V}{\partial y} = (2+3h)r_0^{-h-1}y.$$

For stable equilibrium we must have here

$$2 + 3h < 0.$$

MISCELLANEOUS QUESTIONS.

EDITED BY R. D. CARMICHAEL.

NEW QUESTIONS.

19. How many known proofs are there of the proposition that the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides? Where are the proofs to be found?

20. Some of our readers would like to have a simple account, without proofs, of just what has been accomplished toward the proof of the theorem that the equation $x^n + y^n = z^n$ is impossible in integers when $n > 2$.

21. For the diophantine equation

$$x^2 - y^3 = 17$$

there are known the following solutions:

$$\begin{array}{ccccccccc} x = & 3, & 4, & 5, & 9, & 23, & 282, & 375, & 378661, \\ y = & -2, & -1, & 2, & 4, & 5, & 43, & 52, & 5234. \end{array}$$

One of our readers, who supplied the foregoing facts, desires to know the answers to the following questions: Are there other solutions of the given diophantine equation? How may all the solutions of this equation be found by a systematic procedure?

REPLY.

14. In the process of solving a certain physical problem Professor H. S. Uhler, of Yale University, was led to the definite integral

$$\int_0^a (a^2 - x^2)x dx \int_{a-x}^{a+x} \frac{e^{-cy}}{y} dy,$$

for which he found the value

$$\frac{1}{c^2}\left[a^2 - \frac{3a}{c} + \frac{1}{c}\left(a + \frac{3}{2c}\right)(1 - e^{-2ac})\right],$$

a and c being positive constants. Professor Uhler would like to see how other persons attack the problem of evaluating this integral.

I. Remarks by A. M. Harding, University of Arkansas.

Let

$$u = \int_0^a (a^2 - x^2)xf(x)dx,$$

where

$$f(x) = \int_{a-x}^{a+x} \frac{e^{-cy}}{y} dy.$$

Integrating by parts we have

$$u = -\frac{(a^2 - x^2)^2}{4} f(x)\Big|_0^a + \frac{1}{4}\int_0^a (a^2 - x^2)^2 f'(x)dx;$$

or,

$$u = \frac{1}{4}\int_0^a (a^2 - x^2)^2 f'(x)dx.$$

Now,

$$f'(x) = \frac{e^{-c(a+x)}}{a+x} + \frac{e^{-c(a-x)}}{a-x} = \frac{e^{-ac}}{a^2 - x^2}[a(e^{cx} + e^{-cx}) + x(e^{cx} - e^{-cx})].$$

Therefore

$$u = \frac{ae^{-ac}}{4}\int_0^a (a^2 - x^2)(e^{cx} + e^{-cx})dx + \frac{e^{-ac}}{4}\int_0^a (a^2x - x^3)(e^{cx} - e^{-cx})dx$$

$$= \frac{ae^{-ac}}{4} u_1 + \frac{e^{-ac}}{4} u_2, \text{ say.}$$

Repeated integration by parts gives

$$u_1 = \frac{a^2 - x^2}{c}(e^{cx} - e^{-cx})\Big|_0^a + \frac{2}{c}\int_0^a x(e^{cx} - e^{-cx})dx$$

$$= \frac{2}{c^2}\left[a(e^{ac} + e^{-ac}) - \frac{1}{c}(e^{ac} - e^{-ac})\right].$$

$$u_2 = \frac{a^2x - x^3}{c}(e^{cx} + e^{-cx})\Big|_0^a - \frac{1}{c}\int_0^a (a^2 - 3x^2)(e^{cx} + e^{-cx})dx$$

$$= -\frac{1}{c^2}\left[(a^2 - 3x^2)(e^{cx} - e^{-cx})\Big|_0^a + 6\int_0^a x(e^{cx} - e^{-cx})dx\right]$$

$$= -\frac{1}{c^2}\left[-2a^2(e^{ac} - e^{-ac}) + \frac{6}{c}\left\{a(e^{ac} + e^{-ac}) - \frac{1}{c}(e^{ac} - e^{-ac})\right\}\right]$$

$$= \frac{1}{c^2}\left[2a^2(e^{ac} - e^{-ac}) - \frac{6a}{c}(e^{ac} + e^{-ac}) + \frac{6}{c^2}(e^{ac} - e^{-ac})\right].$$

Substituting the values of u_1 and u_2 in the expression above for u we obtain the desired result.

NOTES AND NEWS.

EDITED BY W. DEW. CAIRNS.

"The propagation of electric waves in wireless telegraphy" is the title of a paper by Professor George R. Dean, of the Missouri School of Mines, in *The Electrician* for September 11, 1914.

Dr. L. L. DINES, formerly professor of mathematics at the University of Arizona, has been elected to an associate professorship at the University of Saskatchewan.

The Southwestern Section of the American Mathematical Society held its eighth regular meeting at the University of Nebraska on Saturday, November 28, 1914.

Mr. J. L. RILEY, formerly instructor in mathematics at the University of Oklahoma, has accepted a fellowship in Rice Institute, Houston, Texas, for the year 1914–15. Professor E. P. R. DUVAL has returned to his former position as associate professor of mathematics at the University of Oklahoma.

Dr. E. E. Whitford has been promoted to an assistant professorship of mathematics at the College of the City of New York.

Mr. Joel D. Eshleman, A.B., has been appointed instructor in mathematics in Adelbert College, Western Reserve University.

Dr. E. B. Stouffer, formerly instructor in mathematics in the University of Illinois, has accepted an assistant professorship in the University of Kansas.

Professor M. Frechet, University of Poitiers, France, who was to lecture at the University of Illinois during the present year, has joined the French army.